

Homework 5

Math 121A, Fall 2025.

Due: Wednesday, Oct 1, 11:59 PM

Unless otherwise stated, all solutions require clear, well-written justification. You must explicitly credit any collaborators for each problem.

The listed problems may be found in Boas's book (third edition).

Problem 1. Recall *Laplace's equation*

$$\Delta f = 0,$$

which if $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, this is

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0.$$

Solutions to Laplace's equation are called *harmonic*.

- (a) Verify that $f = e^x \cos y$ is harmonic.
- (b) One can show that solutions f to Laplace's equation must be smooth, meaning they have all derivatives. This means that mixed partial derivatives should be equal: verify that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$.
- (c) Verify that the function $f = -\log(x^2 + y^2)$, defined everywhere except $\{0\}$ is harmonic by expressing it in polar coordinates (r, θ) and checking that it satisfies Laplace's equation in polar coordinates:

$$\frac{\partial^2 f}{\partial r^2} + \frac{1}{r} \frac{\partial f}{\partial r} + \frac{1}{r^2} \frac{\partial^2 f}{\partial \theta^2} = 0.$$

- Problem 2.** (a) Problem 4.5.1
(b) Problem 4.5.7

Problem 3. Problem 4.6.4

- Problem 4.** (a) Problem 4.7.2
(b) Problem 4.7.7

Problem 5. Problem 4.8.6

- Problem 6.** (a) Problem 4.9.1
(b) Problem 4.9.4
(c) Problem 4.9.11

- Problem 7.** (a) Problem 4.10.1
(b) Problem 4.10.11